MECHANICS 3 (A) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1.	(a) K.E. $= \frac{1}{2}mv^2 = \frac{1}{2}ml^2\omega^2$ J	M1 A1	
	(b) P.E. $= mgl(1 - \cos\theta)$ J	M1 A1	
	(c) Total energy $E = \frac{1}{2}ml^2\omega^2 + mgl(1-\cos\theta)$ J	M1 A1	
	$\frac{dE}{dt} = mgl\sin\theta\frac{d\theta}{dt} = mgl\omega\sin\theta$ Maximum when $\theta = \frac{\pi}{2}$	A1	7
2.	(a) $x = p\sin\omega t + q\cos\omega t$ $\dot{x} = p\omega\cos\omega t - q\omega\sin\omega t$	M1 A1	
	$\ddot{x} = -p\omega^2\sin\omega t - q\omega^2\cos\omega t = -\omega^2(p\sin\omega t + q\cos\omega t) = -\omega^2 x$	M1 A1	
	Acc. prop. to displacement and directed towards O, so SHM	B1	
	(b) Maximum speed at O, so $15 = a\omega = 3a$ $a = 5$ m	M1 A1	7
3.	(a) $T = 0{\cdot}2(1{\cdot}15)\pi^2$, $T = \frac{\lambda}{1}(0{\cdot}15)$, so $\lambda = \frac{(0{\cdot}2)(1{\cdot}15)\pi^2}{0{\cdot}15} = 15{\cdot}1$ N	M1 A1 M1 A1 M1 A1	
	(b) $T_1 = mg = 0{\cdot}2(9{\cdot}8)$ $1{\cdot}96 = \frac{\lambda}{1}x$ $x = 0{\cdot}1298 \text{ m} \approx 13$ cm	B1 M1 A1	9
4.	(a) u = speed before impact. Energy : $3mgl = \frac{3mg}{4l}l^2 + \frac{1}{2}mu^2$ (1)	M1 A1 A1	
	eu = speed after impact. Energy : $2mgl = \frac{3mg}{4l}l^2 + \frac{1}{2}me^2u^2$ (2)	B1 M1 A1	
	(1) gives $u^2 = \frac{9gl}{2}$, so $\frac{5}{4} = \frac{9e^2}{4}$ $9e^2 = 5$ $e = \frac{1}{3}\sqrt{5}$	M1 A1 A1	
	(b) Assumption : P is a particle ; no air resistance	B1	10
5.	(a) When distance from surface $= x$ m, $mv\frac{dv}{dx} = mg - v^2$	B1 M1 A1 A1	
	(b) $\int \frac{mv}{mg - v^2}dv = x + c$ $\frac{1}{2}m\ln(mg - v^2) = x + c$	M1 A1 M1 A1	
	$v = 0, x = 0 : c = -\frac{1}{2}m\ln mg$ $x = \frac{1}{2}m\ln\frac{mv}{mg - v^2}$	M1 A1 A1	
	When $v = \sqrt{\frac{mg}{2}}$, $x = \frac{1}{2}m\ln 2$	M1 A1	13
6.	(a) P moves up $r\sin\theta$ while Q moves down by arc length $r\theta$	M1 A1 A1	
	Ratio of vertical distances moved $= \frac{r\theta}{r\sin\theta} = \frac{\theta}{\sin\theta}$	A1	
	(b) $mg\sin\theta - R = \frac{mv^2}{r}$ Energy : $mgr\theta - mgr\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$	M1 A1 A1 M1 A1 A1	
	$R = mg(2\sin\theta - \theta) = mg(1 - \frac{\pi}{6})$ when $\theta = \frac{\pi}{6}$	M1 A1 A1	13
7.	(a) $M(CD) : \pi r^2 h.\frac{h}{2} - \frac{1}{3}\pi r^2 h.\frac{3h}{4} - \frac{2}{3}\pi r^3.\frac{3r}{8} = \left(\frac{2}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)\bar{y}$	M1 A1 A1 A1	
	$\frac{h^2}{2} - \frac{h^2}{4} - \frac{r^2}{4} = \frac{2}{3}(h+r)\bar{y}$ $\frac{1}{4}(h^2 - r^2) = \frac{2}{3}(h+r)\bar{y}$	M1 A1 A1	
	$\bar{y} = \frac{3}{8}(h - r)$ Modulus needed, since h may be $< r$ or $> r$	A1 B1	
	(b) (i) $\tan\theta = \frac{3}{8}\frac{(h-r)}{r}$ Hemisphere alone : $\tan\theta = \frac{3}{8}$	M1 A1 A1	
	Thus $h - r = r$, so $h = 2r$ $h : r = 2 : 1$	A1	
	(ii) C. of mass must be at O, so $y = 0$, i.e. $h = r$ $h : r = 1 : 1$	M1 A1 A1	16

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